# Zero-Gravity Atmospheric Flight by Robust Nonlinear Inverse Dynamics

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## Nomenclature

$D,L$ = drag and lift forces
$I_{yy}$ = pitch inertia moment
$M$ = pitching moment
$m$ = aircraft mass
$q$ = body-axis pitch angular rate
$V$ = aerodynamic velocity
$\alpha$ = angle of attack
$\gamma$ = flight slope
$\delta_m$ = elevator deflection
$\delta_T$ = throttle setting
$\theta$ = pitch angle
$\tau$ = thrust coefficient

## I. Introduction

ZERO-GRAVITY flights can be obtained for short periods with common subsonic aircraft. Thus, many valuable experiments and otherwise expensive training can be achieved at low cost. This Note considers the design of an autopilot flight control law to realize such a maneuver with a high degree of accuracy. Accuracy is impaired by atmospheric turbulence, local aerodynamic perturbations, and thrust fluctuations on one side and by the limited knowledge about the aircraft mechanics that is used in the design of the control laws on the other side. The control problem dealt with here is highly nonlinear; however, the longitudinal dynamics of an aircraft present[1,2] a particular input-output structure that can be used to get, by inversion, a nominal control law. A sliding controller is added as a regulator to this nominal control law so that the tracking error resulting from perturbations remains uniformly bounded and arbitrarily small after a finite interval of time.

## II. Dynamics of Flight

In this study, it is supposed that the aircraft remains in a vertical plane and that its throttle setting is maintained constant while performing the zero-gravity maneuver. Here the dynamics of flight reduce to

$$\dot{V} = -(D/m) - g\sin(\gamma) \tag{1}$$

$$\dot{\gamma} = \left[L - mg\cos(\gamma)\right]/(mV) \tag{2}$$

$$\dot{\alpha} = q - \left[L - mg\cos(\gamma)\right]/mV \tag{3}$$

$$\dot{q} = M/I_{yy} \tag{4}$$

A thrust coefficient $\tau$ can be introduced to inbed the thrust into the nondimensional aerodynamic coefficients used in the calculation of forces and moments:

$$\tau = \eta(P_{\max}/\bar{q}\sigma V)\delta_T$$

where $\eta$ is the propeller efficiency, $P_{\max}$ is the maximum power of the engine, $\bar{q} = (\frac{1}{2}\rho V^2)$ is the dynamic pressure, and $\sigma$ is a reference area.

It is supposed here that the nondimensional aerodynamic coefficients take the semilinear form:

$$c_i = c_{i0}(\alpha,\beta,\tau) + c_{im}(\alpha,\beta,\tau)\delta_m \tag{5}$$

with $i = M,D,L$ where the $c_{ij}$ are supposed to be smooth functions. Then

$$L = \tfrac{1}{2}\rho\sigma V^2 c_l, \qquad D = \tfrac{1}{2}\rho\sigma V^2 c_d, \qquad M = \tfrac{1}{2}\rho\sigma l V^2 c_M$$

The flight equations must be completed by the conditions

$$\alpha_{\min} \leq \alpha \leq \alpha_{\max} \quad \text{and} \quad \delta_{\min} \leq \delta_m \leq \delta_{\max} \tag{6}$$

The first one guarantees the validity of the aerodynamical model considered and avoids stalling. The second one gives the range of variation of the deflection angle of the elevator and may be completed by a speed condition

$$|\dot{\delta}_m| \leq \dot{\delta}_{m_{\max}} \tag{7}$$

## III. Zero-Gravity Flight

The zero-gravity flight of interest is relative to the body-axis normal load factor at the center of gravity of the aircraft $n_z$ since zero-gravity experiments have to be carried out in this local reference (the aircraft structure) along which it is easier to measure microaccelerations. The body-axis normal load factor $n_z$ calculated at the center of gravity can be expressed as

$$n_z = \left[L\cos(\alpha) + D\sin(\alpha)\right]/mg \tag{8}$$

or now

$$n_z = (\rho\sigma V^2/2mg)c_Z$$

with

$$c_Z = c_l\cos(\alpha) + c_d\sin(\alpha)$$

Thus, the zero-gravity flight is characterized by the following equilibrium condition:

$$c_Z(\alpha,\tau,\delta_m) = 0 \tag{9}$$

Since we suppose that during the maneuver the throttle setting remains constant, then this equilibrium condition can be written as

$$c_{Z_0}\left[\alpha,\tau(V)\right] + c_{Z_1}\left[\alpha,\tau(V)\right]\delta_m = 0 \tag{10}$$

so that the elevator deflection must be chosen as

$$\delta_m = \delta_m(\alpha,V) = -\frac{c_{Z_0}\left[\alpha,\tau(V)\right]}{c_{Z_1}\left[\alpha,\tau(V)\right]} \tag{11}$$

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In the case where the normal factor is calculated at the pilot position, we have

$$n_z = (\rho\sigma V^2/2mg)c_z + (lpM/gI_{yy}) \tag{12}$$

where $lp$ is the distance of the pilot seat to the aircraft center of gravity.

If the wind-axis normal load factor $n_{za}$ is considered, simpler calculations are necessary since in that case

$$n_{za} = \cos(\gamma) + (V/g)\dot{\gamma} \tag{13}$$

and the zero-gravity flight condition reduces to

$$c_l(\alpha,\tau,\delta_m) = 0 \tag{14}$$

## IV. Nonlinear Inverse Control Law

We now consider the problem of driving the aircraft from a constant velocity horizontal flight with $n_z = 1$ to the zero-gravity flight. This transient motion must be achieved smoothly and it is required here that the output of interest $n_z$ follows first-order dynamics such that

$$\tau\dot{n}_z + n_z = n_{z_c} \tag{15}$$

where $\tau$ is chosen conveniently.

Following the technique of nonlinear inverse dynamics,[1] a state representation of the vertical flight dynamics is considered. Choosing the state vector $x' = [V,\gamma,\alpha,q,\delta_m]$, we get the representation

$$\dot{x} = f(x) + hu \tag{16}$$

$$y = c(x) \tag{17}$$

with $u = \dot{\delta}_m$ and $y = n_z$, where

$$f(x)' = [f_1,f_2,f_3,f_4,f_5], \qquad h' = [0,0,0,0,1]$$

$$c(x) = (\rho\sigma V^2/2mg)c_Z(\alpha,V), \qquad f_1 = -D/m - g\sin(\gamma)$$

$$f_2 = [L - mg\cos(\gamma)]/mV, \qquad f_3 = q - [L - mg\cos(\gamma)]/mV$$

$$f_4 = M/I_{yy}, \qquad f_5 = 0$$

and $\dot{\delta}_m$ must be chosen such as

$$\dot{\delta}_m = -\left[\frac{\partial c}{\partial x}f\right]\Big/\left[\frac{\partial c}{\partial x}h\right] + (1/\tau)(n_{z_c} - n_z)\Big/\left[\frac{\partial c}{\partial x}h\right] \tag{18}$$

with

$$\frac{\partial c}{\partial x}h = \left[\frac{\partial L}{\partial \delta_m}\cos(\alpha) + \frac{\partial D}{\partial \delta_m}\sin(\alpha)\right]\Big/mg$$

This term remains different from zero while the effectiveness of the elevator is not null. In the previous relation, it appears that the commanded angular speed of the elevator will be smaller with a higher effectiveness. It is clear that for given incidence and thrust, this effectiveness increases with speed. Also, a longer time constant $\tau$ will imply in a reduced commanded angular speed for the elevator.

To insure that during the maneuver $\alpha$ and $\delta_m$ remain in feasible domains, the initial state and the time constant $\tau$ must be chosen adequately. The maneuver ends at time $t^*$ when it is no longer possible to simultaneously satisfy conditions

$$n_z = 0, \qquad \delta_{\min} \le \delta_m \le \delta_{\max} \qquad \text{and} \qquad \alpha_{\min} \le \alpha \le \alpha_{\max}$$

At this time a new target value $n_{z_c}$ must be chosen to disengage the aircraft from this flight condition.

## V. Robust Regulator

During the zero-gravity flight, the complete dynamics of the (center of gravity and body-axis) system are given by

$$\dot{x} = f(x) + hu + \Delta f(x) + e(t) \tag{19}$$

where $\Delta f(x)$ represents uncertainty or unmodeled dynamics in the model, while $e(t)$ is an external disturbance vector. Thus, the dynamics of the output are

$$\dot{n}_z = \frac{\partial c}{\partial x}\dot{x} = H(x) + G(x)u + E(x,t) \tag{20}$$

$$H(x) = \frac{\partial c}{\partial x}f \tag{21}$$

$$G(x) = \frac{\partial c}{\partial x}h \tag{22}$$

$$E(x,t) = \frac{\partial c}{\partial x}\left[\Delta f(x) + e(t)\right] \tag{23}$$

Choosing $u$ such that

$$u = -H(x)/G(x) + \delta u \tag{24}$$

these dynamics reduce to

$$\dot{n}_z = G(x)\delta u + E(x,t)$$

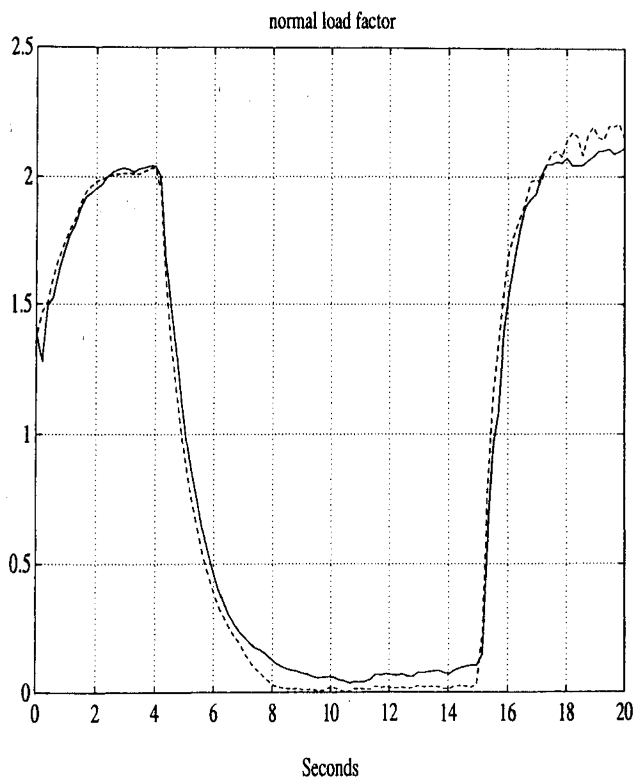


**Fig. 1 Load factor with air turbulence.**

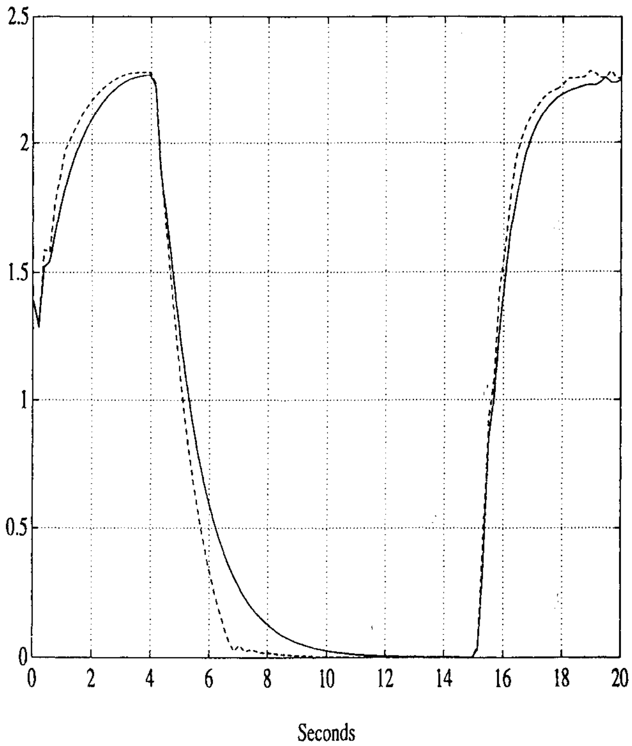


**Fig. 2 Load factor with mass uncertainty (5%).**

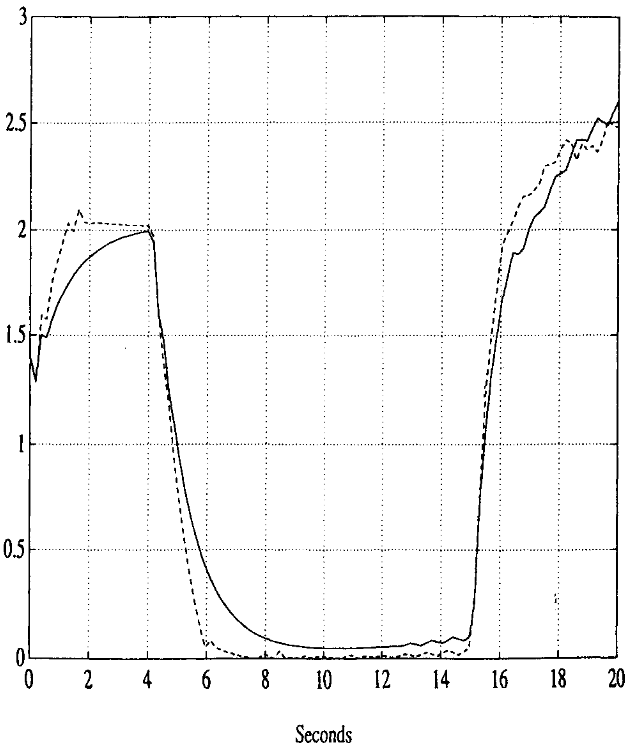


**Fig. 3 Load factor with thrust error.**

A sliding controller[3] can be introduced to regulate the output around the switching surface $n_z(x)=0$ to ensure that the following condition holds after a singular perturbation:

$$n_z \dot{n}_z \leq -\lambda |n_z| \qquad \text{with} \quad \lambda > 0 \tag{25}$$

Thus, $n_z$ goes to zero in a finite time and remains null. The regulator law could be such that

$$\delta u = -k \operatorname{sgn}(n_z)/G(x) \tag{26}$$

where $k$ is taken in accordance with the magnitude of $E(x,t)$.

Let $E_{\max}$ be a bound of $E(x,t)$:

$$|E(x,t)| \leq E_{\max}$$

then, choosing $k = E_{\max} + \lambda$, relation (25) is guaranteed. However, the previous control law is discontinuous across surface $n_z(x)=0$ and leads to high dynamics for the regulator input. These high dynamics, or "chattering," may excite unmodeled structural modes or be counterproductive considering the actuator time delays and dynamics.

Following Slotine,[4] a saturation function may be introduced to avoid this chattering, and the regulator law becomes

$$\delta u = -k \operatorname{sat}_B(n_z)/G(x)$$

with

$$\operatorname{sat}_B(x) = x/B, \qquad \text{if} \quad |x| \leq B$$

and

$$\operatorname{sat}_B(x) = \operatorname{sgn}(x), \qquad \text{if} \quad |x| > B$$

Thus, outside the boundary layer of thickness $2B$, relation (25) stands and if at initial time $t_0$, $|n_z(t_0)| > B$, within a finite time inferior to $|n_z(t_0)|/\lambda$, $n_z$ reaches the boundary layer and stays inside it from then on. Within the boundary layer, the regulator dynamics are given by

$$\tau(t,k,B)\delta\dot{u} + \delta u = e(t,B)$$

with

$$\tau(t,k,B) = 1/(k/B + \dot{G}/G)$$

and

$$e(t,B) = E(t)k/(B\dot{G} + kG)$$

where

$$\frac{\partial \tau}{\partial B} \geq 0 \qquad \text{and} \qquad \frac{\partial}{dB}\left(|e(t,B)|\right) \leq 0$$

Let $\tau_{\min}(k,B) = 1/\max_{X,t}(k/B + \dot{G}/G)$, the regulator bandwidth is bounded by $1/\tau_{\min}(k,B)$ and thus increases with a smaller $B$. Thus, a tradeoff between precision and bandwidth must be solved.

## VI. Simulation Results

The proposed control law has been applied in a simulation study to a single-powered Trinidad aircraft (the TB20 from Socata, a subsidiary of Aerospatiale). An Air Data Package from the fixed-based TB20 training simulator of ENAC has been used in a simulation model with an Adams-Bashforth (order two) method of integration and a 20-ms timestep. Three classes of uncertainties have been considered:

1) Air turbulence $w$ which has been generated using the model of Dryden. In this case it appears, see Fig. 1, that a

large-accuracy improvement is obtained by the use of the sliding controller while all limitations are satisfied.

2) Mass uncertainty: The initial mass of the class of aircraft considered (including the crew) is hardly known with a precision better than 1%, whereas during the flight this mass varies continuously. In this case, the use of the combined nonlinear inverse control and sliding control leads to an increase in the duration of the zero-gravity flight, as shown in Fig. 2.

3) Finally, thrust uncertainty has been considered: A white noise process has been added to a constant bias with respect to the nominal thrust used in the calculation of the control input. Here again, (see Fig. 3) the sliding controller improves both accuracy and duration of the zero-gravity flight.

## References


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